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LETTER TO THE EDITOR

On $1/f$ power spectra

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Abstract. Several works have recently reported $1/f$ noise in the number of particles in some physical systems where white noise is imposed on the boundary. We show that these results should not hold in general because they were *not* derived with a correct definition of the power spectrum.

$1/f$ noise in condensed matter has been considered a puzzling phenomenon by physicists for many years. For a review, see e.g. [3] and [15]. There has recently been an upsurge of interest in this phenomenon after the suggestion by Bak, Tang and Wiesenfeld [1] (BTW) that self-organized criticality might constitute a rather general explanation to the ubiquity of fractal structures in space and time, since $1/f$ noise implies temporal self-similarity. Self-organized criticality is the property of some dissipative many-body systems to evolve naturally towards a metastable, statistically stationary state where events take place at all length and time scales, resulting in fractal structures and $1/f$ power spectra. The paradigmatic example proposed by BTW was a sandpile, which was later shown to exhibit $1/f^2$ noise and not $1/f$ noise as originally claimed [8–10]. In the spirit of self-organized criticality, Jensen [6, 7] suggested that a simple, linear diffusion equation could exhibit a $1/f$ spectrum in the number of particles present in the system, if it were provided with a boundary noise term instead of the usual bulk noise term. Grinstein *et al* [5] later showed that most nonlinearities consistent with the symmetries of the system are irrelevant to the power spectrum. They considered a D-dimensional system in which particles are injected stochastically at a boundary, move into the medium and are removed at the other end. They studied the evolution of the density of particles $N(x,t)$ at location x at time t, and proposed the following stochastic equation:

$$\frac{\partial N(x,t)}{\partial t} = -\nabla J(N) + \delta(x_{\parallel}) J_0(x,t) \qquad (1)$$

where $J(N)$ is the current associated with the local conservation of particles and J_0 is the rate at which particles are injected at $x_{\parallel} = 0$, where $x_{\parallel}$ denotes the direction parallel to the current ($x_{\perp}$ will denote the perpendicular hyperplane). The total number of particles in the system is $N(t) = \tilde{N}(q=0,t)$, where $\tilde{N}(q,t)$ is the spatial Fourier transform of $N(x,t)$. The form of the flow on the entering edge must prevent the density from growing without bounds (in an infinite system), which is crucial, since one would otherwise trivially get $1/f^2$ noise, *the well known result for the Wiener process.* If $J(x,t)$ is first taken in the

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simple mean-field form $-D\,\mathrm{grad}\,N(x,t)$, then a short analysis shows that one apparently gets a $1/f$ form for $\langle|\tilde{N}(f)|^2\rangle$, where $\tilde{N}(f)$ is the Fourier transform of the total number of particles $N(t)$ and angular brackets denote average on many realizations of the process. *Grinstein et al then proved that nonlinear parts of J, neglected in the mean field analysis,* do not affect the form of $\langle|\tilde{N}(f)|^2\rangle$. But we argue here that $\langle|\tilde{N}(f)|^2\rangle \propto 1/f$ does not imply that the power spectrum of $N(t)$ is $1/f$. *In effect, the power spectrum of a time series, be it deterministic or stochastic, is not in general the average square modulus of its Fourier transform.*

In particular, in the process of proving that one should prevent the density from growing without bounds in order to get a non-trivial power spectrum in the number of particles, the authors of [5] though they eventually reached the correct result, used an incorrect argument. In effect, they took $L=\infty$ (where L is the size of the system in the $x_\parallel$ direction), and $J_0(x_\perp,t)=\eta(x_\perp,t)$, where the correlations of the noise term are given by $\langle\eta(x_\perp,t)\eta(x'_\perp,t')\rangle = 2\Gamma\delta(x_\perp-x'_\perp)\delta(t-t')$. They then argued that the equation of motion for the number of particles being

$$\frac{\partial N}{\partial t} = \eta(k_\perp=0,t) \tag{2}$$

(where $k_\perp$ denotes the wavevector associated with $x_\perp$) it follows that the Fourier transform of N is given by $\tilde{N}(f)\propto\tilde{\eta}(k_\perp=0,f)/\mathrm{i}f$, and thus that its fluctuations are given by

$$\langle|\tilde{N}(f)|^2\rangle \propto \frac{\langle|\tilde{\eta}(k_\perp=0,f)|^2\rangle}{f^2} \tag{3}$$

yielding the famous $1/f^2$ spectrum of the Wiener process. They implicitly assumed that $\langle|\tilde{\eta}(k_\perp=0,f)|^2\rangle$ is a constant, but this does not seem to us to be true: if

$$\langle\eta(x_\perp,t)\eta(x'_\perp,t')\rangle = 2\Gamma\delta(x_\perp-x'_\perp)\delta(t-t') \tag{4a}$$

then the correlations of the Fourier transform are given by

$$\langle\eta(k_\perp,f)\eta(k'_\perp,f')\rangle = 2\Gamma\delta(k_\perp+k'_\perp)\delta(f+f'). \tag{4b}$$

Thus, expression (2) yields

$$\langle|\tilde{N}(f)|^2\rangle \propto \frac{\delta(f)}{f^2} \tag{5}$$

which is not a well-defined distribution. Therefore, the $1/f^2$ result for the Wiener process cannot be derived from the average square modulus of its Fourier transform. This is a consequence of the more general remark that *the power spectrum of a signal is not given by the average square modulus of its Fourier transform.* This can be seen from a comparison between the two following definitions which are obviously incompatible (when the expressions exist):

$$\langle|\tilde{N}(f)|^2\rangle = \lim_{T\to\infty}\left\langle\left|\int_{-T}^{T} N(t)\mathrm{e}^{2\mathrm{i}\pi ft}\,\mathrm{d}t\right|^2\right\rangle \tag{6a}$$

$$S_N(f) = \lim_{T\to\infty}\frac{1}{2T}\left\langle\left|\int_{-T}^{T} N(t)\mathrm{e}^{2\mathrm{i}\pi ft}\,\mathrm{d}t\right|^2\right\rangle. \tag{6b}$$

Expression (6*a*) gives the average square modulus of the Fourier transform of a signal N, while expression (6*b*) is one among several possible equivalent definitions of the power spectrum of N. When both expressions exist, there may be a relationship between them, but no such general relationship can be defined *a priori*.

Along the same lines, the authors of [12] arrived at the correct result relying on the same type of mistake in proving that the BTW sandpile [1] exhibits $1/f^2$ noise in the total number of particles in the system: they argued that the fluctuations $h(x,t)$ of the height of the sandpile could be described by

$$\frac{\partial h}{\partial t} = -\nabla \cdot j + \eta(x,t) \tag{7}$$

where j is the current of particles and η is a non-conserving noise with correlations given by equation (4*a*). Integrating equation (7) over space yields an equation similar to equation (2): $\partial N/\partial t = \eta(k=0,t)$ because the current vanishes at the border. They conclude that the power spectrum of N is $1/f^2$ (which is correct), because

$$S_h(f) \propto \langle|\tilde{h}(f)|^2\rangle \propto \frac{\langle|\tilde{\eta}(k=0,f)|^2\rangle}{f^2}$$

(which we argued to be false). Thus, although expression (26) in [12] is false, the correct solution was eventually reached because common wisdom holds that the Wiener process trivially yields a $1/f^2$ power spectrum. We shall briefly rederive this result in the next section.

To see how all this affects the results of [4–7], we first briefly recall a few facts about the power spectrum of a process, and then turn to the simple model proposed by Jensen [7].

The power spectrum of a *second-order stationary process* $X(t,\mu)$ (with μ a realization of the process), i.e. having constant expectation $\langle X(t)\rangle = E$, and an autocorrelation function $C_{xx}(t,t+\tau) = \langle X(t)X(t+\tau)\rangle$ depending only on time difference τ, can be defined as the Fourier transform of its autocorrelation function, when it exists [2, 14]:

$$S_x(f) = \int_{-\infty}^{+\infty} C_{xx}(\tau)\mathrm{e}^{2\mathrm{i}\pi f\tau}\,\mathrm{d}\tau \tag{8}$$

where $C_{xx}(\tau)$ is the autocorrelation of $X(t)$:

$$C_{xx}(\tau) = \langle X(t)X(t+\tau)\rangle. \tag{9}$$

The Wiener–Khinchin theorem relates this definition to the periodogram $P_T(f)$ of the truncated signal

$$X_T(t,\mu) = \begin{cases} X(t,u) & \text{if } -T \leqslant t \leqslant +T \\ 0 & \text{if } |t| > T \end{cases}$$

$$P_T(f) = \frac{1}{2T}\left|\int_{-T}^{T} X(t,\mu)\mathrm{e}^{2\mathrm{i}\pi ft}\,\mathrm{d}t\right|^2. \tag{10}$$

Making the average over all possible realizations μ and *then* taking the limit when $T \to \infty$, one gets:

$$S_x(f) = \lim_{T\to\infty}\langle P_T(f)\rangle. \tag{11}$$

This theorem is valid if

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\tau| C_{xx}(\tau) e^{-2i\pi f\tau} \, d\tau = 0. \tag{12}$$

Condition (12) is almost always satisfied in practice, but most physicists prefer to use the definition involving the autocorrelation function. For instance, it is easy to show that the autocorrelation function of a Langevin process is of the form $C_{xx}(\tau) = e^{-a|\tau|}$ (obviously obeying (12)), where $1/a$ is a (positive) characteristic time, so that the resulting power spectrum is the Lorentzian form

$$S_x(f) = \frac{2a}{a^2 + 4\pi^2 f^2}. \tag{13}$$

A lot of 'explanations' of $1/f$ noise in condensed matter rely on the idea that superposed Lorentzian spectra with an appropriate weighting function can generate $1/f$ noise [3, 15]. By $1/f$ noise, one usually refers to signals exhibiting a power spectrum of the form $1/f^a$, with $0 < a < 2$. Because experimental signals are usually of finite length, there is a cutoff at small frequencies preventing infrared divergence for $a \geqslant 1$, and because such signals are of finite resolution, there is also a cutoff at high frequencies, preventing ultraviolet divergence for $a \leqslant 1$. While signals having $a < 1$ are stationary, *signals with $a > 1$ are generally non-stationary*, the case $a = 1$ usually being considered as the boundary between stationary and non-stationary signals [16], though the issue remains somewhat controversial. For non-stationary signals, equation (11) should be the relevant definition of the power spectrum. *The power spectrum, however, is not very informative for such signals*, and the analysis should be made within the context of time-frequency methods. Anyway, if one takes, for instance, a Wiener process, considered as the integral of a white noise process $Y(t)$, one finds, using expression (11):

$$\begin{aligned}
S_x(f) &= \lim_{T\to\infty} \langle P_T(f)\rangle = \lim_{T\to\infty} \frac{1}{T} \left\langle \left| \int_0^T e^{ift} \int_0^t Y(u)\, du \right|^2 \right\rangle \\
&= \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_0^T dt\, dt' e^{if(t-t')} \int_0^t du \int_0^t du' \delta(u-u') \\
&= \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_0^T dt\, dt' e^{if(t-t')} \min(t, t') \\
&= \lim_{T\to\infty} \frac{2}{T} \mathrm{Re} \left[\int_0^T dt \int_0^t dt' e^{if(t-t')} \right] \\
&= \lim_{T\to\infty} \frac{2}{T} \mathrm{Re} \left[i \int_0^T dt e^{ift} \frac{\partial}{\partial f} \left\{ \int_0^t dt' e^{-ift'} \right\} \right] \\
&= \frac{2}{f^2}
\end{aligned} \tag{14}$$

which is the correct result, sometimes convenient to use. One must remember that theoretical Brownian motion is not physical (white noise has infinite energy, and 'practical' white noise is slightly coloured). For non-stationary processes, the power spectrum can also be defined *in principle* with the help of the autocorrelation function assuming that integral and expectation signs can be switched:

$$S_x(f) = \int_{-\infty}^{+\infty} d\tau\, e^{-2i\pi f\tau} \left[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} C_{xx}(t, t+\tau)\, dt \right] \tag{15}$$

where C_{xx} now has two arguments since X is no longer stationary. For instance, remembering that for the Wiener process, which is one of the best-known examples of a non-stationary process, $C_{xx}(s,t) = \min(s,t)$, one finds that $S_x(f) \propto 1/f^2$, with a multiplicative constant K. The strict application of (15) leads to K diverging, but for any practical purpose, with data of finite length and resolution, the integral over τ has finite bounds. Expression (15) applies well to, and is most often used for, cyclostationary signals, where the autocorrelation function is periodic.

The usual diffusion equation assumes that particles are subjected to random encounters with other particles, the effects of which can be modelled by a white noise term in the bulk. The resulting motion is an unbounded Brownian motion (the Wiener process) and thus results in a $1/f^2$ power spectrum. Jensen [7] proposed that the very same diffusion equation for $N(r,t)$ the particle density

$$\frac{\partial N(r,t)}{\partial t} = \gamma \nabla^2 N(r,t) \tag{16}$$

with *white noise boundary conditions* for $N(r,t)$ would yield a $1/f$ spectrum in any dimension for the total number of particles $N(t) = \int N(r,t)\,\mathrm{d}^D r$, where D is the spatial dimension. His proof can be sketched as follows: if $P(r,t) = B(r,t)$ for r belonging to the boundary S, the solution for $P(r,t)$ is given by

$$N(r,t) = -\frac{1}{4\pi}\int \mathrm{d}t_0 \int \mathrm{d}S_0 B(r_0,t_0)\nabla_0 G(r,r_0 \mid t,t_0) \tag{17}$$

where G is the Green function solution of the equation

$$\frac{\partial G}{\partial t} - \gamma\nabla^2 G = 4\pi\delta(r-r_0)\delta(t-t_0) \tag{18}$$

with $G(r,r_0 \mid t,t_0) = 0$ for r_0 belonging to S. If the domain is sufficiently large, the Green function can be approximated by $G(r,t) = (4\pi\gamma)^{1-D/2}t^{-D/2}\mathrm{e}^{-r^2/4\gamma t}\Theta(t)$, so that

$$\int \mathrm{d}^D r \nabla G(r,t) \approx t^{-1/2}. \tag{19}$$

Expression (19) is in fact valid only when the number of particles is determined in the vicinity of the boundary, as was already the case in a $1/f$ (transient-diffusion) model described in [11]. In this case, for a large domain, one has

$$\tilde{N}(f) = \tilde{N}(q=0,f) \cong \int \mathrm{d}t_0 \int \mathrm{d}S_0 B(r_0,t_0)\mathrm{e}^{-2\mathrm{i}\pi f t_0} \int \mathrm{d}t \int \mathrm{d}^D r \nabla G(r,t)\mathrm{e}^{-2\mathrm{i}\pi f t} \tag{20}$$

where $\tilde{N}(f)$ is the Fourier transform of $N(t)$, and $\tilde{N}(q,f)$ is the Fourier transform in space and time of $N(r,t)$. The product of the two terms in expression (20) comes from the fact that $N(q=0,t)$ is the *approximate convolution of two terms*, so that the Fourier transform yields a simple product. Now, as we argued in the introduction, the expression

$$|\tilde{N}(q=0,f)|^2 \approx \left|\int \mathrm{d}t^{-1/2}\mathrm{e}^{2\mathrm{i}\pi f t}\right|^2 \sim \frac{1}{f} \tag{21}$$

is not well defined if one assumes that the boundary factor in (20) is a white noise term: $|\tilde{N}(q=0,f)|^2$ does not exist. Moreover, expression (21) simply does not define the power spectrum of the process. The power spectrum is, assuming that $N(t)$ is zero for $t < 0$, defined by

$$S_N(f) = \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_0^T \mathrm{d}t N(t) \mathrm{e}^{2\mathrm{i}\pi f t} \right|^2 \right\rangle \tag{22}$$

which is clearly different. In the particular case studied here, although expression (21) is not correct, one effectively gets a $1/f$ noise. In effect, let us assume that $N(t)$ is the (approximate for a large but finite system) convolution of two functions h and g: $N = h * g$ (h being, for instance, the white noise term).

$$\begin{aligned}
S_N(f) &= \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_{-T}^{+T} \mathrm{d}t\, \{h^* g\}(t) \mathrm{e}^{2\mathrm{i}\pi f t} \right|^2 \right\rangle \\
&= \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_{-T}^{+T} \mathrm{d}t\, \mathrm{e}^{2\mathrm{i}\pi f t} \int_{-\infty}^{+\infty} \mathrm{d}t'\, g(t')h(t-t') \right|^2 \right\rangle \\
&= \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_{-\infty}^{+\infty} \mathrm{d}t' g(t') \int_{-T}^{+T} \mathrm{d}t \mathrm{e}^{2\mathrm{i}\pi f t} h(t-t') \right|^2 \right\rangle \\
&= \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_{-\infty}^{+\infty} \mathrm{d}t' \mathrm{e}^{2\mathrm{i}\pi f t'} g(t') \int_{-T-t'}^{+T-t'} \mathrm{d}u \mathrm{e}^{2\mathrm{i}\pi f u} h(u) \right|^2 \right\rangle \\
&\approx \left\{ \lim_{T\to+\infty} \frac{1}{T}\left\langle \left| \int_{-T-t'}^{+T-t'} \mathrm{d}u \mathrm{e}^{2\mathrm{i}\pi f u} h(u) \right|^2 \right\rangle \right\} \left\langle \left| \int_{-\infty}^{+\infty} \mathrm{d}t' \mathrm{e}^{2\mathrm{i}\pi f t'} g(t') \right|^2 \right\rangle \\
&\approx C \left\langle \left| \int_{-\infty}^{+\infty} \mathrm{d}t' \mathrm{e}^{2\mathrm{i}\pi f t'} g(t') \right|^2 \right\rangle
\end{aligned} \tag{23}$$

where C is a constant, because $\{\lim_{T\to+\infty}(1/T)\langle| \int_{-T-t'}^{+T-t'} \mathrm{d}u \mathrm{e}^{2\mathrm{i}\pi f u} h(u)|^2|\rangle\}$ is precisely the (constant) power spectrum of the white noise term, not to be confused with $\langle| \int_{-\infty}^{+\infty} \mathrm{d}u \mathrm{e}^{2\mathrm{i}\pi f u} h(u)|^2\rangle$ which is not defined. Since here $g(t) \propto t^{-1/2}$ (given by expression (19)), the power spectrum is given by $S_N(f) \approx C\langle| \int_{-\infty}^{+\infty} \mathrm{d}t' \mathrm{e}^{2\mathrm{i}\pi f t'} g(t')|^2\rangle \propto 1/f$. Thus the $1/f$ result holds despite the incorrectness of its derivation in [7].

It is more difficult to evaluate the consequences of the incorrect definition on the model studied in [5]. The conclusion to be drawn is that nothing can be said analytically about the power spectrum of the signal generated by the number of particles in the system using the same approach as the authors of [5]. In effect, their result comes from the approximate equality

$$\tilde{N}(k = 0, f) \propto \frac{\eta(k_\perp = 0, f)}{f^{1/2}} \tag{24}$$

which is supposed to imply

$$\langle|\tilde{N}(k = 0, f)|^2\rangle \propto \left\langle \left| \frac{\eta(k_\perp = 0, f)}{f^{1/2}} \right|^2 \right\rangle. \tag{25}$$

But even if one replaces $\langle|\tilde{N}(k = 0, f)|^2\rangle$ with the right definition of the power spectrum, since expressions (24) and (25) are not well-defined, it is difficult to derive anything about the true power spectrum. In particular, their claim that this model generates a $1/f$ noise requires more investigation.

We now consider numerical estimates. The formula shown explicitly in [4] to calculate the power spectrum still contains the same mistake. In effect, the approximate numerical power spectrum of the discrete time series generated by the total number of particles $N(t)$ in the system should be given by

$$S_x(f) = \frac{1}{T}\left\langle \left| \sum_{t=1}^{T} N(t) e^{2i\pi f t} \right|^2 \right\rangle \tag{26}$$

where the $1/T$ term is not only a numerical factor: considering it as a simple numerical factor leads to it being neglected [4] and to defining the power spectrum as the square modulus of the Fourier transform of the signal, which, once again, it is not. This problem can be seen by rescaling formula (26):

$$S_x(f) = \frac{f}{T}\left\langle \left| \sum_{t=1}^{T/f} N(t) e^{2i\pi f t} \right|^2 \right\rangle \tag{27}$$

so that the smaller the frequency, the more points are used to calculate the corresponding power. It can be seen from equation (11) that $P_T(f)$ is an estimator of $P(f)$, and more precisely that $P_T(f)$ is the power spectrum of the known part $X_T(t)$ of the signal in the interval $[-T, +T]$. The inverse Wiener–Khinchin theorem implies that $P_T(f)$ is the Fourier transform of an associated autocorrelation function $C_{xx}^T(\tau)$ which is itself an estimator of C_{xx} : $C_{xx}^T(\tau)$ converges to $C_{xx}(\tau)$ for any τ, but the convergence is not uniform, and the variance of $C_{xx}^T(\tau)$ is small for small τ only, while it increases as $|\tau| \to T$.

In conclusion, one should remember that the power spectrum of a signal is not in general given by the average square-modulus of the Fourier transform of the signal. We have found that the confusion of both quantities is present in many papers during the last few years. Such confusion does not always lead to errors in the results, because the preferred definition of the power spectrum seems most often to be the one involving the autocorrelation function of the signal. Yet, as noticed in [12], it is easier, and more appropriate for non-stationary signals, to compute the power spectrum from the periodogram as in equations (10), (11) (as we argued in the previous section it is also a more dangerous method, from the numerical point of view). Moreover, stationarity is too often assumed when computing the power spectrum from the autocorrelation function: although expression (15) constitutes an alternative way of defining the spectral density in non-stationary cases, it is not always applicable, and equations (10) and (11) remain safer. Finally, let us point out that our letter does not question the results of [8–10] which prove that the power spectrum of the number of particles in the BTW sandpile is a $1/f^2$ spectrum, because the calculations performed in these papers rely on the autocorrelation function.

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